MULTIPLE CHOICE Circle the correct answers. **SCORE: _____ / 4 PTS**

[1] Which of these is/are improper ? #1➡ $\int_0^{\frac{\pi}{4}} \cot x \, dx$ #2➡ $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec x \, dx$ #3➡ $\int_{-3}^{-1} \frac{1}{x^2+5x-6} \, dx$

CIRCLE THE LETTER BELOW WHICH CORRESPONDS TO THE CORRECT ANSWER.

[A]	none	[B]	only #1	[C]	only #2	[D]	only #3
[E]	only #1 and #2	[F]	only #1 and #3	[G]	only #2 and #3	[H]	all

2

[2] Which of these converge(s) ? #1➡ $\int_0^1 \frac{1}{\sqrt{x}} \, dx$ #2➡ $\int_1^\infty \frac{1}{x^\pi} \, dx$ #3➡ $\int_0^\infty \frac{1}{\pi^x} \, dx$

CIRCLE THE LETTER BELOW WHICH CORRESPONDS TO THE CORRECT ANSWER.

[A]	none	[B]	only #1	[C]	only #2	[D]	only #3
[E]	only #1 and #2	[F]	only #1 and #3	[G]	only #2 and #3	[H]	all

2

Give **very** brief answers. Explanations are **not** required. **SCORE: _____ / 4 PTS**

[1] If $h(x) \le x^{-3}$ for all x, what can you conclude about $\int_1^\infty h(x) \, dx$? NOTHING 2

[2] If $f(x) \ge g(x) \ge 0$ for all x, and $\int_0^\infty f(x) \, dx$ diverges, what can you conclude about $\int_0^\infty g(x) \, dx$? NOTHING 2

Evaluate $\int_1^\infty xe^{-3x} \, dx$. If the integral diverges, write "DIVERGES". **SCORE: _____ / 6 PTS**

$= \lim_{N\to\infty} \left(-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}\right)\Big|_1^N$ (1½)

$= \lim_{N\to\infty} \left(-\frac{1}{3}Ne^{-3N} - \frac{1}{9}e^{-3N} - \left(-\frac{1}{3}e^{-3} - \frac{1}{9}e^{-3}\right)\right)$ (1½)

$= 0 - 0 + \frac{4}{9}e^{-3}$

$= \frac{4}{9}e^{-3}$ (1½)

u	dv
x	e^{-3x}
1	$-\frac{1}{3}e^{-3x}$
0	$\frac{1}{9}e^{-3x}$

(1½) $\lim_{N\to\infty} Ne^{-3N} = \lim_{N\to\infty} \frac{N}{e^{3N}} = \lim_{N\to\infty} \frac{1}{3e^{3N}} = 0$

Evaluate $\int 6x^2 \ln(1+4x^2)\,dx$.

SCORE: ____ / 5 PTS

$= 2x^3 \ln(1+4x^2) - \int \frac{16x^4}{1+4x^2}\,dx$

$= 2x^3 \ln(1+4x^2) - \int \left(4x^2 - 1 + \frac{1}{1+4x^2}\right) dx$

$= 2x^3 \ln(1+4x^2) - \frac{4}{3}x^3 + x - \frac{1}{2}\tan^{-1} 2x + C$

① POINT EACH

u	dv
$\ln(1+4x^2)$	$6x^2$
$\frac{8x}{1+4x^2}$	$2x^3$

$$\begin{array}{r} 4x^2 - 1 \\ 4x^2+1 \overline{)\,16x^4} \\ \underline{16x^4 + 4x^2} \\ -4x^2 \\ \underline{-4x^2 - 1} \\ 1 \end{array}$$

Evaluate $\int \frac{121 - t}{(t+2)(t^2 - 6t + 25)}\,dt$.

$= \int \left(\frac{A}{t+2} + \frac{B(2t-6) + C}{(t-3)^2 + 16}\right) dt$

SCORE: ____ / 5 PTS

$A[(t-3)^2 + 16] + B(2t-6)(t+2) + C(t+2) = 121 - t$ ①

$t = -2$: $41A = 123 \rightarrow A = 3$

$t = 3$: $16A + 5C = 118 \rightarrow 5C = 70 \rightarrow C = 14$

COEF OF t^2: $A + 2B = 0 \rightarrow B = -\frac{1}{2}A = -\frac{3}{2}$

SANITY CHECK: $t = 2$ $\quad \frac{119}{4(17)} \stackrel{?}{=} \frac{3}{4} + \frac{-\frac{3}{2}(-2) + 14}{17}$

$\frac{7}{4} \stackrel{?}{=} \frac{3}{4} + \frac{17}{17} = \frac{7}{4}$ ✓

$= \int \left(\frac{3}{t+2} + \frac{-\frac{3}{2}(2t-6) + 14}{(t-3)^2 + 16}\right) dt$ (½ ½ ½ ½)

$= \frac{1}{2}\,3 \ln|t+2| - \frac{3}{2}\ln(t^2 - 6t + 25) + \frac{7}{2}\tan^{-1}\frac{t-3}{4} + C$ (½ ½ ①)

Evaluate $\int \frac{t - 9}{(t+3)(t^2 + 2t - 3)}\,dt$.

$= \int \frac{t-9}{(t+3)(t+3)(t-1)}\,dt$

$= \int \left(\frac{A}{t+3} + \frac{B}{(t+3)^2} + \frac{C}{t-1}\right) dt$ (①)

SCORE: ____ / 6 PTS

$A(t+3)(t-1) + B(t-1) + C(t+3)^2 = t - 9$ (1½)

$t = 1$: $16C = -8 \rightarrow C = -\frac{1}{2}$

$t = -3$: $-4B = -12 \rightarrow B = 3$

COEF OF t^2: $A + C = 0 \rightarrow A = \frac{1}{2}$

SANITY CHECK: $t = 2$ $\quad \frac{-7}{5(5)} \stackrel{?}{=} \frac{\frac{1}{2}}{5} + \frac{3}{25} + \frac{-\frac{1}{2}}{1}$

$\frac{-7}{25} \stackrel{?}{=} \frac{1}{10} + \frac{3}{25} - \frac{1}{2} = \frac{5 + 6 - 25}{50} = \frac{-14}{50} = \frac{-7}{25}$ ✓

$= \int \left(\frac{\frac{1}{2}}{t+3} + \frac{3}{(t+3)^2} - \frac{\frac{1}{2}}{t-1}\right) dt$ (½ ½ ½)

$= \frac{1}{2}\ln|t+3| - \frac{3}{t+3} - \frac{1}{2}\ln|t-1| + C$ (½ ① ½)